Quantum mechanics in a nut-shell

- each **quantum mechanical system** is associated to a Hilbert **space** *H* (of **wave functions**)

- each dynamical **variable** (observable) is associated to a **hermitian operator** Ĉ acting on elements of *H*
  measurable values (observables) are **eigenvalues** of Ĉ (discrete or continuous)

  ***NOTE**: time is a parameter → no operator to MEASURE time;*
  indirectly from temporal evolution of other operators

- Schrödinger equation (SEQ) (*) → partial differential equation

$$i\hbar \, (\delta/\delta t) \, \Psi = \hat{H} \, \Psi$$

  Ĥ from Hamilton function via correspondence principle

time-independent SEQ

$$\hat{H} \, \Psi_n = E_n \, \Psi_n$$

Ĥ contains total energy of the system

wave-functions / states Ψ_n = eigenvectors of Ĥ
total energy in a state Ψ_n = eigenvalue E_n of Ĥ

time-dependent SEQ

$$i\hbar \, (\delta/\delta t) \, \Psi(t) = \hat{H} \, \Psi(t)$$

$\Psi(t)$ superposition of Ψ_n

most relevant quantum mechanical systems

free particle $E = \frac{\hbar^2 k^2}{2m}$ planes waves, wave packets

particle in a box $E_n \sim \frac{n^2}{L^2}$

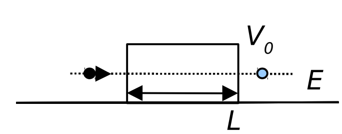
harmonic oscillator (parabolic potential V=kx²) $E_n \sim \hbar\omega(n + \frac{1}{2})$ Hermite polynoms

coulomb potential, H atom $E_n \sim \frac{Z^2}{n^2}$ (2l+1) x degenerate

$$\Psi_{nlm} = R_{nl}(r) \times Y_{lm}(\theta, \phi)$$

spherical harmonics $Y_{lm}(\varphi,\theta)$, Laguerre polynoms
3 quantum numbers {n,l,m}

tunnelling $T \sim e^{-\frac{2}{\hbar}\sqrt{2m(V_0 - E)}L}$



(*) *valid for pure states, for mixed states: PDE for density operator*

approximations – perturbation theory $\hat{H} = \hat{H}_0 + \hat{H}_{int}$ ($\hat{H}_{int} \ll \hat{H}_0$)

time-independent PT

$$E_n^{(1)} = \langle \Psi_m^0 | \hat{H}_{int} | \Psi_n^0 \rangle$$

$$\Psi_n^{(1)} = \sum_m \frac{\langle \Psi_m^0 | \hat{H}_{int} | \Psi_n^0 \rangle}{E_m^0 - E_n^0} \Psi_m^0$$

time-dependent PT

for $\hat{H}_{int}$ periodic or constant in time

$$k_{if} = \frac{2\pi}{\hbar} |\langle \Psi_i | \hat{H}_{int} | \Psi_f \rangle|^2 \delta(E_i - E_f)$$

(FERMIs Golden Rule)

→ transition rates

symmetries

$\hat{S}$ is a symmetry operator if $[\hat{H},\hat{S}] = 0$
$\hat{H}$ and $\hat{S}$ have common set of eigenvectors

for each system $\hat{H}$ several $\hat{S}_i$ can exist
each eigenvector fully determined by *set* of eigenvalues of $\hat{H}$ and *all* $\hat{S}_i$

properties of $\hat{S}$ can be used to solve SEQ

$\hat{S}$ leads to a preserved quantity (NÖTHER theorem)
eigenvalues ↔ “good quantum numbers”

spin of particle determines how it interacts with identical particles

half-numbered spin:	**fermions**	Fermi distribution (**PAULI principle**)
integer spin:	**bosons**	Bose Einstein distribution

spin cannot be derived within non-relativistic quantum mechanics

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Ψ(t) superposition of Ψn

most relevant systems

free particle

particle in a box

harmonic oscillator

coulomb potential, H atom

tunnelling

approximations – perturbation theory

time-independent PT

symmetries $[\hat{H},\hat{S}] = 0$

Ĥ and Ŝ have common set of eigenv
(with different eigenvalues, to giver

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Quantum mechanics in a nut-shell

- each **quantum mechanical system** is associated to a Hilbert **space** *H* (of **wave functions**)
- each dynamical **variable** (observable) is associated to a **hermitian operator** $\hat{C}$ acting on elements *H*
  measurable values (observables) are **eigenvalues** of $\hat{C}$ (discrete or continuous)

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- Schrödinger equation (SEQ) (*) -> partial differential equation

$$i\hbar \, (d/dt) \, \Psi_n = \hat{H} \, \Psi_n$$

  $\hat{H}$ from Hamilton function via correspondence principle
  bound states: $E_n < 0$

time-independent SEQ

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