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H.G. Evertz,* E. Katznelson, P.G. Lauwers,
and M. Marcu*

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Abstract

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*II. Institut für Theoretische Physik, Luruper Chaussee 149, 2000 Hamburg 50, Federal Republic of Germany

Post address:
Nussallee 12
D-5300 Bonn 1
W-Germany

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FINITE-SIZE EFFECTS IN THE $SU(2)$ HIGGS MODEL: A NUMERICAL INVESTIGATION

H. G. Evertz†, E. Katznelson‡, P. G. Lauwers‡ and M. Marcu†

† II. Institut für Theoretische Physik,
Luruper Chaussee 149, 2000 Hamburg 50,
Federal Republic of Germany

‡ Physikalisches Institut der Universität Bonn
Nußallee 12, 5300 Bonn 1,
Federal Republic of Germany

ABSTRACT

We present new results from a Monte Carlo simulation of the Higgs region of the $SU(2)$ gauge theory with a fundamental scalar matter field. To investigate the model at $\lambda = \infty$, $\beta = 2.4$ and 2.7, and κ close to the phase transition, three different lattice sizes were used: $8^3 \times 24$, $10^3 \times 24$ and $12^3 \times 24$. The finite size effects, which are by no means negligible, are discussed in detail.

1. INTRODUCTION

The nonperturbative properties of the Glashow-Weinberg-Salam model of weak and electromagnetic interactions[1] are extremely important from a purely theoretical as well as from a phenomenological viewpoint. Analytic methods that lead to concrete information about these properties have not been developed yet. The most promising technique has been the Monte-Carlo (MC) simulation of the lattice regularized model. There are however severe conceptual[2] and technical[3] problems in treating the Standard Model fermions on the lattice. On the other hand, today's computers can well handle the problem of gauge fields coupled to scalar matter fields. Before starting a large scale simulation of the full $SU(2) \times U(1)$ gauge-Higgs system, it is important to gain a good quantitative understanding of the simpler $SU(2)$ and $U(1)$ theories separately.

Here we report on a high-statistics simulation of the $SU(2)$ gauge-Higgs system with the Higgs field in the fundamental representation. Many aspects of this model have been investigated before by other groups, both by simulations[4,5,6,7] and by perturbative methods[8]. From a numerical point of view the model is not an easy one. It has three independent parameters, β, κ, and λ. A complete numerical investigation should include simulations for the full range of these parameters for large enough lattices. While providing an overall picture, the previous simulations often underestimated (or neglected) the dependence of the results on the lattice size. One of the aims of our Monte Carlo study is to deal with the finite size effects in a systematic way.

Some preliminary results for $\beta = 3.0$ and $\lambda = \infty$ were reported on before [9]. We had found considerable finite-size effects in the particle masses for lattices up to $12^3 \times 24$. In order to study these effects in more detail we decided to perform simulations at $\lambda = \infty$ and at smaller values of β, where the W mass is larger. For κ we chose values in the Higgs region ($\kappa > \kappa_{crit}$) close to the phase transition. The strategy is to move in a controlled way towards the region of larger β, where the renormalized weak coupling constant takes values compatible with the experimental one[7].

In section 2 we briefly review the model and the physical quantities we studied. In section 3 we describe the MC simulations and results. Some concluding remarks are presented in section 4.

2. THE MODEL AND THE MEASURED QUANTITIES

The $SU(2)$ Higgs model, as the $SU(2)$ lattice gauge theory with a Higgs field in the fundamental representation is commonly called, is defined by the action[4-7,9]:

$$S = -\frac{\beta}{2}\sum_P Tr\, U_P - \kappa \sum_{x,\mu} \rho_x\, \rho_{x+\hat{\mu}}\, Tr(\sigma_x^\dagger\, U_{x,\mu}\, \sigma_{x+\hat{\mu}}) + \lambda \sum_x (\rho_x^2 - 1)^2 + \sum_x \rho_x^2. \quad (1)$$

Here $U_{x,\mu}$ is the $SU(2)$ element on the link starting from the lattice point $x = (\underline{x}, t)$ in the μ direction, U_P is the usual plaquette variable, $\rho_x = (\phi_x, \phi_x)$ is the length of the two-dimensional complex scalar field ϕ_x, and σ_x is the $SU(2)$ matrix that rotates the vector $(1,0)$ into ϕ_x/ρ_x. As usual, we denote the gauge invariant link variable by

$$V_\mu(x) = \sigma_x^\dagger\, U_{x,\mu}\, \sigma_{x+\mu} = V_{0,\mu}(x) + i\sum_{r=1}^{3} V_{r,\mu}(x)\, \tau_r, \quad (2)$$

where τ_r are the usual Pauli matrices. Besides the gauge symetry the model has an $SU(2)$ global "isospin" symmetry (conjugation of $V_\mu(x)$ by a constant matrix). The indices 0 and r in (2) denote the isospin zero part and the three components of the isospin one part.

2.1. The W and Higgs masses

To determine the W mass we consider the decay of the correlation function[5-7]

$$K_W(T) = \left\langle \sum_{r=1}^{3}\sum_{j=1}^{3} \left[\sum_{\underline{x}} V_{r,j}(\underline{x},0)\right] \left[\sum_{\underline{y}} V_{r,j}(\underline{y},T)\right] \right\rangle, \tag{3}$$

where j denotes the three spacelike directions and the summation $\sum_{\underline{x}}$ runs over all three-dimensional space coordinates. For the computation of the Higgs mass an appropriate correlation function is

$$K_H(T) = \left\langle \left[\sum_{\underline{x}}\sum_{j=1}^{3} V_{0,j}(\underline{x},0)\right] \left[\sum_{\underline{y}}\sum_{j=1}^{3} V_{0,j}(\underline{y},T)\right] \right\rangle - \left\langle \left[\sum_{\underline{x}}\sum_{j=1}^{3} V_{0,j}(\underline{x},0)\right] \right\rangle^2 . \tag{4}$$

Because of the periodic boundary conditions, the measured correlation functions (3) and (4) are fitted by a periodic exponential decay in T (the fit parameters are A and m):

$$K(T) = A\left\{\exp[-am\,T] + \exp[-am\,(L_t - T)]\right\}. \tag{5}$$

Here a is the lattice spacing, m is the mass in physical units, and L_t is the lattice size in time direction. Eq. (5) holds for T large, but $T < L_t/2$.

2.2. Static potential, renormalized gauge coupling and source energy

Several interesting physical quantities can be derived by studying the $R \times T$ space-time Wilson loops $W(R,T)$ in the fundamental representation.

The potential $V_W(R)$ between two fundamental representation static sources at distance R is obtained from the exponential decay in time of $W(R,T)$. At fixed R a two-parameter ($C_W(R)$ and $V_W(R)$) fit is performed ($R < T < L_t/2$):

$$W(R,T) = C_W(R)\,\exp[-V_W(R)\,T\,]. \tag{6}$$

After $V_W(R)$ is determined in this way for $1 \leq R \leq L_s/2$ (L_s is the lattice size in space direction), this function of R is compared to the Yukawa potential $Y(R,am_W)$ obtained in the one-W-exchange approximation. For this we do a two-parameter (α_{ren} and E_q) fit:

$$V_W(R) = -\frac{3\alpha_{ren}}{4}\,Y(R,am_W) + 2E_q. \tag{7}$$

Here either the continuum Yukawa form, $Y_{cont}(R,am_W) = \exp(-\,am_W\,R)/R$, or a lattice version $Y_{latt}(R,am_W)$ [6,7], which takes into acount the correct range for the momenta on the finite lattice, can be used. Since this potential is derived as a first perturbative approximation (one W, no loops), it cannot be expected to fit the data well for all values of β, κ, and λ, and for the full range of R. If the fit is good, then α_{ren} is equal to $g_{ren}^2/4\pi$ with g_{ren} the renormalized gauge coupling, and E_q is the energy of a fundamental static source. Depending on the version of the Yukawa potential used, we will write $E_{q,cont}$ and $\alpha_{ren,cont}$, or $E_{q,latt}$ and $\alpha_{ren,latt}$.

In principle one could fit the potential with m_W as a third free parameter, instead of using the value obtained from eq. (5). In practice, however, this is notoriously difficult[5,6,7,9] since for the range of available R-values the fits are rather insensitive to changes in m_W.

The static source energy E_q can also be determined in two other ways. The first method uses the expectation values of gauge invariant timelike lines:

$$G_1(T) = \left\langle \frac{1}{2} Tr \left[\sigma^{\dagger}_{(\underline{x},0)} \left(\prod_{t=0}^{T-1} U_{(\underline{x},t),0} \right) \sigma_{(\underline{x},T)} \right] \right\rangle . \tag{8}$$

For large values of T but with $T < L_t/2$, a two-parameter (C_1 and $E_{q,line}$) fit is performed:

$$G_1(T) = C_1 \exp(-E_{q,line}T). \tag{9}$$

The second alternative consists of directly computing the source energy from the measured expectation value of the timelike Polyakov loop $P_t \equiv G_1(L_t)$ by means of the relation

$$E_{q,Pol} = -(1/L_t) \ln(P_t). \tag{10}$$

These different methods for computing the same physical quantity provide an excellent way of checking the consistency of measurements and computational methods.

2.3. The Vacuum Overlap Order Parameter (VOOP)

In the continuum theory it is customary to consider the Higgs expectation value $v =< \phi^c >$ (ϕ^c is the continuum Higgs field), which is related to the Fermi coupling constant. This quantity v cannot be defined unless the gauge is fixed. A better quantity to consider is the VOOP. It has a clear gauge-independent physical interpretation[10]. In addition to the Wilson loops $W(R,T)$, another set of gauge-independent quantities $G_2(R,T)$ must be measured:

$$G_2(R,T) = \left\langle \frac{1}{2} Tr \left[\sigma^{\dagger}_{(\underline{x},0)} \left(\prod_{t=0}^{T-1} U_{(\underline{x},t),0} \prod_{r=0}^{R-1} U_{(\underline{x}+r\hat{1},T),1} \prod_{t=T-1}^{0} U^{\dagger}_{(\underline{x}+R\hat{1},t),0} \right) \sigma_{(\underline{x}+R\hat{1},0)} \right] \right\rangle \tag{11}$$

The VOOP is then defined as the limit $R, T \to \infty$ of the function

$$\rho(R,T) = \frac{G_2(R,T)}{W(R,2T)^{1/2}} \tag{12}$$

with $T \geq cR$ (c is a positive constant). It can be shown that for all gauges in the tree-level approximation the VOOP $\equiv \rho(\infty,\infty)$ is related to v by the expression

$$(av)^2 = \kappa\, \rho(\infty,\infty). \tag{13}$$

In the next section we will refer to the VOOP determined in this way as VOOP_{dir}.

It is not always an easy task to determine this limit numerically in the way described before, especially in the case of small lattices. A good way to avoid these difficulties is to take the limit $T \to \infty$ first, while keeping R fixed at a finite value. We can do this for the numerator and the denominator separately by performing two-parameter fits according to eq. (6) and to

$$G_2(R,T) = C_2(R)\exp[-V_{G_2}(R)T]. \tag{14}$$

V_W and V_{G_2} represent the same potential, fitted from two different quantities. If the fits (6) and (14) are both good and after checking that $V_W = V_{G_2}$, we have:

$$\rho(R,\infty) = \frac{C_2(R)}{C_W(R)^{1/2}}. \tag{15}$$

This allows us to determine the VOOP if the lattice is large enough in the space direction. The value obtained in this way will be referred to as VOOP_{rat}.

It has been suggested theoretically that in a phase without free charges the VOOP is equal to the constant C_1 obtained from the fit (9) [10]. This is a numerically easier way to determine the VOOP. As in the case of the source energy, having more than one method to determine a given quantity provides a useful check of the internal consistency of our calculation.

3. THE MONTE CARLO SIMULATIONS AND THE RESULTS

As opposed to the case of pure matter theories, for the $SU(2)$ Higgs model a theory of the finite size effects has not been worked out. The most reliable way to check whether one has these effects under control still is to perform simulations on lattices of increasing size until the changes in the measured quantities become smaller than the statistical errors. For this reason we carried out simulations for three different lattice sizes ($8^3 \times 24$, $10^3 \times 24$, $12^3 \times 24$). We will refer to these sizes as small, medium and large respectively.

We will present data for $\lambda = \infty$, $\beta = 2.4$ and 2.7. For each β we took two values of κ that are in the Higgs phase, relatively close to the phase transition. The MC runs started with approximately 15000 thermalization sweeps, followed by 90000 to 120000 sweeeps during which measurements were performed every third sweep. We used a fully vectorized version[11] of Creutz's heat-bath algorithm[12]. In order to check whether any flip-flops to the confinement phase had occured, the full history of eight quantities was stored. In the subsequent analysis we found no such flip-flops.

We were very careful with the error analysis. By the central limit theorem[13], the averages of the measured physical quantities over a long time series are approximately Gaussian random variables. The Gaussian distribution is characterized by a covariance matrix. One way to estimate it is to partition the time series into bins and compute the covariance matrix for the bin averages. As the bin size is increased, this bin average covariance matrix divided by the number of bins converges to the covariance matrix for the whole-run averages. In computing functions of the measured quantities or in fitting them, we did the error analysis using the whole covariance matrix. We will discuss below how the more widespread procedure of only considering the variances (i. e. the diagonal of the covariance matrix) may lead to numerical mistakes.

Our analysis of the MC results is summarized in tables 1 and 2. Almost all symbols used in these tables were defined in section 2. The last two entries are the values of the W masses as computed from the tree-level relation (16) (see section 3.5). The numbers followed by a * are not as trustworthy as the others. They either result from fits that are not good (reduced $\chi^2 > 1$), or from a numerical limiting procedure where it was not clear that the asymptotic value had really been reached.

3.1. The W and Higgs masses

Fitting the masses with eq. (5) is a delicate task. One has to vary the shortest distance T_{min} and the longest distance T_{max} and find the situations for which the reduced χ^2 is smaller than one. The value of the fitted parameters and their statistical error depends very much on the number of data points considered, especially on T_{min}. The results quoted here are those with the smallest errors among the subset of fits that have $\chi^2 < 1$ and that give stable mass values with respect to

Table 1: Collected results for $\beta = 2.7$

	$\kappa = .365$			$\kappa = .370$		
Lattice size	$8^3 \times 24$	$10^3 \times 24$	$12^3 \times 24$	$8^3 \times 24$	$10^3 \times 24$	$12^3 \times 24$
am_W	.41(2)	.32(2)	.33(2)	.40(2)	.31(1)	.31(1)
am_H	.59(2)	.53(3)	.53(3)	.56(8)	.73(2)	.74(2)
VOOP_{dir}	.101(2)	.101(4)	.097(2)*	.130(2)	.129(2)	.126(2)
VOOP_{rat}	.103(6)	.105(4)	.105(2)	.133(4)	.133(3)	.131(5)
C_1	.101(6)*	.104(4)	.103(3)	.138(5)	.137(2)	.135(1)
$E_{q,line}$	.267(6)*	.251(5)	.243(3)	.257(4)	.242(2)	.235(2)
$E_{q,Pol}$	.277(4)	.258(2)	.243(1)	.256(3)	.241(1)	.234(1)
$E_{q,cont}$	.219(1)	.228(2)*	.228(1)	.218(1)	.225(2)	.224(1)
$E_{q,latt}$	.206(2)	.218(3)	.220(1)	.210(1)	.216(1)	.217(1)
$\alpha_{ren,cont}$	.297(6)	.39(2)*	.36(2)	.296(4)	.37(2)	.37(2)
$\alpha_{ren,latt}$	.279(4)	.36(2)	.34(1)	.276(4)	.34(2)	.32(1)
$am_{W,cont}$	.27(1)	.31(1)*	.29(1)	.30(1)	.34(1)	.33(1)
$am_{W,latt}$	.26(1)	.29(1)	.29(1)	.29(1)	.32(1)	.31(1)

Table 2: Collected results for $\beta = 2.4$

	$\kappa = .390$			$\kappa = .400$		
Lattice size	$8^3 \times 24$	$10^3 \times 24$	$12^3 \times 24$	$8^3 \times 24$	$10^3 \times 24$	$12^3 \times 24$
am_W	.47(2)	.41(2)	.38(2)	.47(1)	.44(1)	.40(2)*
am_H	.80(2)	.55(6)*	.67(4)	.91(3)	.96(3)	.94(3)
VOOP_{dir}	.165(3)	.167(6)	.168(2)	.217(3)	.216(2)	.216(5)
VOOP_{rat}	.172(14)	.168(4)	.171(4)	.216(7)	.215(9)	.215(12)
C_1	.183(3)	.170(3)	.178(2)*	.228(3)	.231(1)	.226(1)*
$E_{q,line}$	.312(3)	.298(2)	.301(2)*	.288(2)	.286(1)	.282(1)*
$E_{q,Pol}$	.306(9)	.301(4)	.297(3)	.290(6)	.281(3)	.279(2)
$E_{q,cont}$	.283(8)*	.293(5)*	.293(4)*	.287(2)	.281(1)	.280(3)
$E_{q,latt}$	.270(5)*	.283(2)*	.284(3)*	.268(1)	.276(1)	.275(2)
$\alpha_{ren,cont}$	.56(12)*	.61(7)*	.58(4)*	.64(4)	.58(3)	.51(8)
$\alpha_{ren,latt}$	.51(11)*	.53(6)*	.50(4)*	.60(3)	.51(3)	.42(7)
$am_{W,cont}$	.48(7)*	.50(3)*	.49(2)*	.59(3)	.56(3)	.52(6)
$am_{W,latt}$	.46(7)*	.47(3)*	.46(2)*	.57(3)	.53(3)	.48(5)

small changes in T_{min} and T_{max}.

For a correct analysis of the goodness of fit, it is absolutely necessary to use the full covariance matrix for the measurements at different values of T. We compared the fits done in this way with those disregarding the covariances between different quantities ,i. e. , using only the variances. For the same T_{min} and T_{max}, the values of the fitted parameters differed only marginally (rarely up to one standard deviation); the errors estimated with the full covariance matrix were usually smaller by 20 to 50%. On the other hand, the value of χ^2 was often larger for the fits that used the full covariance matrix, in some cases by as much as 100 %. Thus if we only use the variances, we may be misled to declare as good a fit starting from a too small T_{min}. In this case it is not only the errors that are not estimated correctly, but also the values of the fitted physical quantities.

For our values of the coupling constants, fitting the Higgs mass is more difficult than fitting the W mass, as the correlation function K_H drops off much faster and becomes zero (within error bars) for distances around 7 lattice units.

The analysis for the data at $\beta = 2.7$ went very smoothly. From table 1 it is clear that the difference in masses between the medium and large lattice size is zero within error bars. It is interesting that $\kappa = 0.365$ and 0.370 give the same W mass. At $\beta = 3.0$[9] there was a similar situation close to the phase transition. There we could not exclude the possibility that this was due to finite-size effects. For $\beta = 2.7$ we are now more confident that the finite size effects are under control. New data for $\beta = 3.0$ on larger lattices, that will be published elsewhere, point in the same direction.

The data for $\beta = 2.4$ are not as good as for $\beta = 2.7$. As expected for a smaller β, the measurements are much "noisier". This makes fits more difficult, especially for the W mass. Again, its values for $\kappa = 0.39$ and 0.40 agree within error bars. On the other hand it is also clear that the results for the medium and large lattice are still quite different. New data on a $14^3 \times 28$ lattice seem to confirm the values quoted for the large lattice.

The Higgs mass varies strongly with κ, as expected from other studies[5,6,7,9]. In our case, m_H is in the region of $2m_W$. It may be that some of the finite size effects are due to the crossover (as the lattice size is changed) from a situation with a stable Higgs to one where the Higgs particle is a resonance.

3.2. Static potential, renormalized gauge coupling and source energy

As described in section 2.2, the fit of the potentials to the measured data is done in two steps. First the potentials $V(R)$ for different spatial distances R are determined together with their covariance matrix. This procedure is similar to the one used for the masses and it always worked smoothly. However, employing the full covariance matrix (of $W(R,T)$) makes an even greater difference here.

The second step, where some potential function is fitted to $V(R)$, is more delicate. One of the reasons is that the correct potential function that should be used to fit the measured data is not known theoretically. This leads to the paradoxical situation that it is often easier to fit potentials to lower statistics data than to high statistics data.

At $\beta = 2.7$ the fits with both versions of the Yukawa potential were satisfactory in all cases except one. For distances R in the range $2 \le R \le L_s/2$ the fits were always good. Furthermore, the results did not change when the values of the potential for $R = 6$ and/or $R = 5$ were excluded. All results quoted in table 1 are for $2 \le R \le 4$. Only in one case ($\kappa = 0.365$, medium lattice, continuum potential) the fit for $3 \le R \le 5$ was quite bad and gave results that were different from the ones quoted in table 1. For this reason we marked this case by a *.

For our data at $\beta = 2.4$ and $\kappa = 0.390$ the fitting of potentials was not successful for two reasons: the data are noisier and the renormalized gauge coupling is quite large. Usually the fit including only distances from 3 to 5 was good, but these results are not very reliable because the fitted parameters are quite insensitive to the data for large distances. If we include the data for $R = 2$ the fits become bad. The explanation is that for these values of the parameters and for these distances the one-particle-exchange approximation breaks down and different potential functions should be used. Previous simulations at similar values of β [6] also found that close to the phase transition the Yukawa fits do not work well. On the other hand, for $\kappa = 0.400$ the fitting was as good as for the two κ values at $\beta = 2.7$. This is probably due to the fact that by increasing κ, am_H increases and α_{ren} decreases[5,6]. In table 2 we have again quoted the results for $2 \le R \le 4$, even for $\kappa = .390$.

The energy E_q of a static source can be computed in four different ways. On a finite lattice with large L_t, $E_{q,Pol}$ and $E_{q,line}$ should be equal. The finite size corrections to $E_{q,cont}$ and $E_{q,latt}$ however should neither be equal to each other, nor to those of $E_{q,Pol}$ and $E_{q,line}$. This different finite size dependence can indeed be seen from our data. For the small lattice the four values for the source energy are usually somewhat apart. At $\beta = 2.4$, $\kappa = .400$, they converge towards compatible values as the lattice size is increased. At $\beta = 2.7$ however, they are still not equal within error bars on the large lattice.

3.3. VOOP

For the small lattice it is very difficult, if not impossible, to obtain a reliable value for VOOP_{dir} because it is not feasible to check whether the asymptotic value has already been reached. However from the data for the large lattices one clearly sees that this limit has already been reached within error bars for $R = 3$ or 4. For this reason the values and errors quoted for the small lattice in the tables are the ones for $R = 3$.

As described in section 2.3 there is a more reliable way to determine the VOOP, by taking at fixed R the limit $T \rightarrow \infty$ for the numerator and the denominator independently (VOOP_{rat}). For the small lattices the subsequent limit $R \rightarrow \infty$ is again problematic but for the larger lattices this procedure gives a stable asymptotic value. A third method to determine the VOOP is to use C_1 (see (9)). In the data C_1 is always larger or equal (within error bars) than the values for the VOOP calculated in the two previous ways. Again the different methods should not have identical finite size effects. For $\beta = 2.7$ they converge towards compatible values. For $\beta = 2.4$ the procedure to determine C_1 was not so successful and the values for C_1 are still larger than VOOP_{dir} or VOOP_{rat}.

It should be noticed that the values that we obtained for VOOP_{rat} hardly change with growing lattice size. This is one additional reason to trust this method more than the other ones.

3.4. Tree-level relations

From our discussion of the VOOP ($\equiv \rho(\infty,\infty)$) it is clear that the gauge invariant two-point function $\rho(R,\infty)$ can be determined numerically in a consistent way in the Higgs region. It is instructive to check the validity of the tree-level relation that gives m_W as a function of $\rho(\infty,\infty)$ and α_{ren}:

$$(am_W)^2 = 2\pi\,\alpha_{ren}\,(a\,v)^2 = 2\pi\,\alpha_{ren}\,\kappa\,\rho(\infty,\infty)\,. \tag{16}$$

In this relation we can use $\alpha_{ren,latt}$ or $\alpha_{ren,cont}$, and the values for m_W are labeled correspondingly. The values obtained in this way were included in the tables.

The results for $\beta = 2.7$ agree quite well numerically with the values for m_W obtained directly, at least for the larger lattices where we do no longer expect strong finite-size effects. A similar result was found for $\beta = 3.0$[9]. This indicates that the renormalized perturbation theory can still be used here. Notice that the bare and the renormalized weak couplings differ considerably at $\beta = 2.7$. Perturbation theory predicts that at fixed β the r. h. s. of (16) grows with $(\kappa - \kappa_{crit})$. This too seems to be reflected in the behavior of the data. As pointed out before, our values for the directly measured m_W show (within error bars) no κ dependence, both at $\beta = 2.7$ and 3.0.

For $\beta = 2.4$, the tree-level relation for the W mass does not hold , even for the large lattice. This should be expected for small values of β.

4. CONCLUSIONS

We performed a simulation in the Higgs region of the SU(2) lattice gauge theory with a fundamental scalar matter field, close to the transition to the confinement region. We investigated the finite size effects for several quantities: masses m_H and m_W, renormalized gauge coupling α_{ren}, source energy E_q and gauge invariant Higgs expectation value defined by the VOOP.

As we had hoped at the outset, the finite size effects at $\beta = 2.7$ are smaller than they had been at $\beta = 3.0$[9], and our results here converge to the values on the $12^3 \times 24$ lattice (with the noteable exception of E_q). Somewhat surprisingly, there are larger finite size effects at the still smaller $\beta = 2.4$.

In the data presented here, and also at $\beta = 3.0$[9], we do not see (within error bars) an increase with κ of the directly measured value of am_W close to the Higgs phase transition.

It is remarkable that the tree level relation (16) is fullfilled quite well at $\beta = 2.7$ (and 3.0) although $\alpha_{ren}/\alpha_{bare}$ is large (2 to 3). The values of α_{ren} itself and of λ_{ren} defined by $\lambda_{ren} = m_H^2/4v^2$ are not too large for the renormalized perturbation theory to be applicable. On the other hand, the values of v as computed from the VOOP are very different from the values of v in the pure matter theory at $\lambda = \infty$ and similar κ[14]. Thus we cannot expect that all corrections due to the gauge coupling can be computed using the tree-level approximation. In order to really compare the simulation with perturbation theory, a one-loop calculation with the results expressed in terms of the renormalized couplings is needed.

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