Simulation of the 3-state Potts model with chemical potential[1]

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Abstract. The 3-state Potts model with chemical potential is mapped to a flux representation where the complex action problem is resolved. We perform a Monte Carlo simulation based on a worm algorithm to study the phase diagram of the model. Our results shed light on the role which center symmetry and its breaking play for the QCD phase diagram.

Keywords: Potts model, QCD phase diagram, sign problem, worm algorithm
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Lattice simulations of QCD with finite density are notoriously difficult due to the complex phase which the fermion determinant acquires at non-vanishing chemical potential μ. Here we explore new strategies using the 3-state Potts model in 3 dimensions with finite μ as an effective theory that includes the leading center symmetric and center symmetry breaking terms. It can be obtained from QCD in the limit of strong coupling and infinite quark mass. Understanding its phase diagram will allow one to judge which role these terms play for the QCD phase diagram. The action is given by

$$-\tau\sum_{\langle x,y\rangle} 2\,\mathrm{Re}\,P(x)P(y)^* - \kappa\sum_x \Big[e^{\mu}P(x) + e^{-\mu}P(x)^*\Big], \quad (1)$$

where the nearest neighbor coupling τ is an increasing function of the temperature and κ a decreasing function of the quark mass. The degrees of freedom are the center phases $P(x) \in \{1, e^{\pm i2\pi/3}\}$ located at the sites x of a hypercubic 3-dimensional lattice. The model has a complex action problem which limited previous simulations [1] to small μ. It can be mapped to a flux representation [2]

$$Z = \sum_{\{d,m\}} W(d,m) \prod_x T\left(\sum_\nu [d_{x,\nu} - d_{x-\hat{\nu},\nu}] + m_x\right). \quad (2)$$

The new degrees of freedom are dimer and monomer variables, $d_{x,\nu}$, $m_x \in \{-1,0,1\}$ on the links and sites of the lattice, respectively. The triality function $T(n)$ is 1 if n is a multiple of 3 and vanishes otherwise. The weight factor $W(d,m)$ is a function of the original couplings τ, κ and μ. It is always real and positive, thus the complex action problem is solved.

We apply a variant of the worm algorithm [3] to the flux representation (2) and study the phase diagram in the τ-μ plane for $\kappa = 0.01$. We determine transition lines from the maxima of different second derivatives of the free energy. They do not exhibit volume scaling. Using histogram techniques we establish that they are lines of smooth crossover. Different second derivatives peak at different parameter values (Fig. 1). The shape of the phase diagram resembles the form suggested for QCD. There, however, one expects that sea quarks which are neglected here turn the crossover region into a single first order line for sufficiently large μ. We expect that the techniques outlined here can be applied to more realistic effective theories of QCD and provide insight into the role of symmetries in the QCD phase diagram.

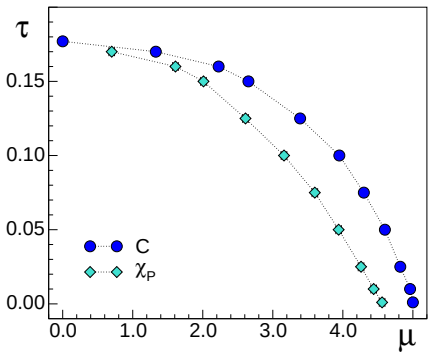

FIGURE 1. Crossover lines in the τ-μ plane at $\kappa = 0.01$ as determined from the maxima of the heat capacity C and the Polyakov loop susceptibility χ_P.

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